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**MATHEMATICS**

**SPECIALIST**

**UNIT 1**

**Semester One**

**2019**

**SOLUTIONS**

***Calculator−free Solutions***

1.



✓ parallelogram with 2v and u as sides

✓ parallelogram with v and –u as sides

✓ vector u in the direction of –v with magnitude |u|

 [6]

2. (a) (i) $\left(\begin{matrix}3\\-5\end{matrix}\right)=k\left(\begin{matrix}4\\α\end{matrix}\right)$ ✓

 $∴k=\frac{3}{4} \rightarrow α=-\frac{5}{k}=-\frac{20}{3}$ ✓

 (ii) $\left(\begin{matrix}4\\α\end{matrix}\right)∙\left(\begin{matrix}1\\1\end{matrix}\right)=0$ ✓

 $∴4+α=0 \rightarrow α=-4$ ✓

 (iii) $\vec{PQ}=\left(\begin{matrix}1\\α+5\end{matrix}\right)$

 since the *x*–coordinate is already 1 unit in length, then

 the y–coordinate must be zero. ✓

 $∴α=-5$ ✓

 (iv) PQ as base ⇒ |OP| = |OQ|

 $\left|\begin{matrix}3\\-5\end{matrix}\right|=\left|\begin{matrix}4\\α\end{matrix}\right| \rightarrow \sqrt{34}=\sqrt{16+α^{2}}$ ✓

 $∴α^{2}=18 \rightarrow α=\pm 3\sqrt{2}$ ✓

2. (b) (i) Solving simultaneously (any method, elimination shown below):

 $ \begin{matrix}u\\v\end{matrix} \begin{matrix}=\\=\end{matrix} \begin{matrix}-6i\\2i\end{matrix} \begin{matrix}-2j\\+3j\end{matrix} \begin{matrix}×3\\×2\end{matrix} \rightarrow \begin{matrix}3u\\2v\end{matrix} \begin{matrix}=\\=\end{matrix} \begin{matrix}-18i\\4i\end{matrix} \begin{matrix}-6j\\+6j\end{matrix} \downright \left(+\right) $

 $∴ 3u+2v=-14i \rightarrow i=-\frac{3}{14}u-\frac{1}{7}v$ ✓✓

 similarly (or by substitution):

 $ \begin{matrix}u\\v\end{matrix} \begin{matrix}=\\=\end{matrix} \begin{matrix}-6i\\2i\end{matrix} \begin{matrix}-2j\\+3j\end{matrix} \begin{matrix}×1\\×3\end{matrix} \rightarrow \begin{matrix}u\\3v\end{matrix} \begin{matrix}=\\=\end{matrix} \begin{matrix}-6i\\6i\end{matrix} \begin{matrix}-2j\\+9j\end{matrix} \downright \left(+\right) $

 $∴ u+3v=7j \rightarrow j=\frac{1}{7}u+\frac{3}{7}v$ ✓✓

 (ii) $r=14\left(-\frac{3}{14}u-\frac{1}{7}v\right)+7\left(\frac{1}{7}u+\frac{3}{7}v\right)$

 $∴r=-2u+v$✓✓ [14]

3. (a) (i) $20!-18!=20×19×18!-18!$ ✓

 $=\left(20×19-1\right)×18!$

 $=\left(380-1\right)k=379k$ ✓

 (ii) $\frac{}{}=\frac{20!}{3!}÷\frac{21!}{3!×18!}$ ✓

 $=\frac{20!}{3!}×\frac{3!×18!}{21×20!}=\frac{k}{21}$ ✓

 (b) RHS $=\left(\begin{matrix}n\\n-r\end{matrix}\right)=\frac{n!}{\left(n-r\right)!×\left[n-\left(n-r\right)\right]!}$ ✓

 $=\frac{n!}{\left(n-r\right)!\left[r\right]!}$ ✓

 $=\left(\begin{matrix}n\\r\end{matrix}\right)=$ LHS [6]

4. (a) If $m<1$, then $m>m^{2}$. ✓

 It is NOT always true because it does not work for negatives. ✓

 e.g. $m=-2<1 \rightarrow m^{2}=4>m ∴$ false ✓

 The converse is always true for $0<m<1$ ✓

 (b) If the parallelogram is not a rectangle, then it does not have congruent diagonals. ✓

 Yes it is always true as only squares and rectangles have congruent diagonals. ✓

 (c) For all rational numbers ✓, there exists two integer numbers $a$ and $b$ ✓

 such that $p$ is the quotient of $a$ and $b$. [8]

5. (a) (i) $2^{6}=1+6+15+20+15+6+1=64$ ✓

 (ii) $11^{5}=\left(10+1\right)^{5}$

 $=10^{5}+5×10^{4}+10×10^{3}+10×10^{2}+5×10+1^{5}$ ✓

 $=100 000+50 000+10 000+1 000+50+1$

 $=161051$ ✓

 (b) (i) $x=3$ since $=20$ ✓

 (ii) $x=7$ since $=21$ ✓

 (iii) $x=8$ since $=$ ✓

 (c) $\left(2x-y\right)^{5}$

 $=\left(2x\right)^{5}+5\left(2x\right)^{4}\left(-y\right)^{1}+10\left(2x\right)^{3}\left(-y\right)^{2}+10\left(2x\right)^{2}\left(-y\right)^{3}+5\left(2x\right)^{1}\left(-y\right)^{4}+\left(-y\right)^{5}$ ✓

 $=32x^{5}-80x^{4}y+80x^{3}y^{2}-40x^{2}y^{3}+10xy^{4}-y^{5}$ ✓✓

 (d) (i) $=56$ ✓

 (ii) $×=1×20=20$ ✓

 (iii) $×+×$ ✓

 $=3×10+1×10=30+10=40$ ✓ [13]

6. (a) $\vec{AD}=\frac{2}{5}\vec{AB}=\frac{2}{5}\left(b-a\right)$ ✓

 $\vec{CD}=\vec{CO}+\vec{OA}+\vec{AD}=-\frac{1}{2}b+a+\frac{2}{5}\left(b-a\right)$ ✓

 $∴\vec{CD}=\frac{3}{5}a-\frac{1}{10}b$✓

 (b) $\vec{OC}+\vec{CE}=\vec{OE} \rightarrow \vec{OC}+β\vec{CD}=α\vec{OA}$ given

 $∴\frac{1}{2}b+β\left(\frac{3}{5}a-\frac{1}{10}b\right)=αa$✓

 $×10 \rightarrow 5b+6βa-βb=10αa$

 $\rightarrow \left(6β-10α\right)a=\left(β-5\right)b$

 since $a$ and $b$ are non–parallel, then:

 $β-5=0 \rightarrow β=5$ ✓

 $6β-10α=0 \rightarrow α=\frac{3}{5}β=3$ ✓ [6]

***Calculator−assumed Solutions***

7. (a) ABC collinear ⇒ AB // BC

 $\vec{AB}=\left(\begin{matrix}-2\\4\end{matrix}\right)-\left(\begin{matrix}4\\-5\end{matrix}\right)=\left(\begin{matrix}-6\\9\end{matrix}\right)$

 $\vec{BC}=\left(\begin{matrix}-6\\10\end{matrix}\right)-\left(\begin{matrix}-2\\4\end{matrix}\right)=\left(\begin{matrix}-4\\6\end{matrix}\right)$

 $∴\left(\begin{matrix}-6\\9\end{matrix}\right)=k\left(\begin{matrix}-4\\6\end{matrix}\right) \rightarrow \begin{matrix}k=\frac{-6}{-4}=\frac{3}{2}\\k=\frac{9}{6}=\frac{3}{2}\end{matrix}$ ✓✓

 Since $k$ is unique, then AB // BC and hence ABC collinear. ✓

 (b) $\left|AB\right|=\left|\begin{matrix}-6\\9\end{matrix}\right|=3\left|\begin{matrix}-2\\3\end{matrix}\right|$ and $\left|BC\right|=\left|\begin{matrix}-4\\6\end{matrix}\right|=2\left|\begin{matrix}-2\\3\end{matrix}\right|$ ✓

 $∴AB:BC=3:2$ ✓ [5]

8. (a) ∠PFO = 35° ✓

 Because ΔOFP is isosceles since |OP| = |OF| = radii ✓

 (b) ∠FEP = 55° ✓

 Since ∠FOP = 110° from ΔOFP, and the angle at the centre

 is double the size of the angle at the edge. ✓

 (c) ∠PQF = ∠FEP = 55° ✓

 Angles at the circumference within the same segment

 are congruent. ✓

 (d) ∠CFP = ∠FEP = 55° ✓

 The alternate segment theorem ✓

 (e) |GC| = |CF| = 11 – |FB| = 11 – 8 = 3 cm ✓

 Tangents to a circle from the same external point

 are congruent. ✓

 (f) |AM|×(|AM| + 2×radius) = |AH|2

|AM|×(|AM|+8) = 52 ✓

 |AM|2+8|AM| – 25 = 0

 CAS ⇒ |AM| = $-4\pm \sqrt{41}$ ✓

 $∴$ |AM| = $\sqrt{41}-4 ≈2.40$ cm only solution ✓ [13]

9. (a) (i) Divisible by 3 and 5 = divisible by 15

 100 ÷ 15 = 6.6 ⇒ only 6 elements are divisible by 15 ✓

 Therefore, assuming every other element is chosen

 instead of those 6, we need 100 – 6 +1 = 95 elements ✓

 (ii) Divisible by 3 = 100 ÷ 3 = 33.3 ⇒ 33 elements

 Divisible by 5 = 100 ÷ 5 = 20 elements ✓

 Divisible by 3 or 5 = 33 + 20 – 6 = 47 elements ✓

 Assuming the other 53 elements are chosen first,

 then 53 + 1 = 54 elements must be chosen ✓

 (b) Assuming the highest numbers are chosen first:

 100 + 99 + 98 + … + 91 + 90 = 955 ✓

 If 89 is chosen next then the sum exceeds 1000. ✓

 Therefore, a maximum of 11 elements must be chosen. ✓ [8]

10. (a) $n\left(M∪C\right)=n\left(M\right)+n\left(C\right)-n\left(M∩C\right)$ ✓

 14 334 ✓ = 7 531 + 9 885 – $n\left(M∩C\right)$

 $∴n\left(M∩C\right)$ = 3 082 households ✓

 (b) $n\left(M∪C∪B\right)=n\left(M\right)+n\left(C\right)+n\left(B\right)$

 $ - n\left(M∩C\right)-n\left(M∩B\right)-n\left(C∩B\right)$

 $+n\left(M∩C∩B\right)$ ✓

 $∴n\left(M∪C∪B\right)=$ 7 531 + 9 885 + 4 977 – 3 082 – 2 252 – 4 310 + 1 724

 = 14 473 that have all three ✓

 Therefore, 16 366 – 14 473 = 1 893 households have neither ✓ [6]

11. (a) (i) $=1 413 720 OR \left(×4!\right)$ ✓

 (ii) $××4!=351 000$ ✓✓

 (iii) $××=22 440$ ✓✓

 (b) II and III ✓✓

 (c) $=×$

 $\frac{\left(x+1\right)!}{\left(x+1-3\right)!}=4×\frac{x!}{\left(x-2\right)!}$ ✓

 $\frac{\left(x+1\right) × x!}{\left(x-2\right)!}=4\frac{x! }{\left(x-2\right)!}$ ✓

 $\left(x+1\right)=4 \rightarrow x=3$ ✓

11. (d) LHS $=\frac{n!}{\left(n-2\right)!}+2n×\frac{\left(n-1\right)!}{\left(n-1\right)!}$ ✓

 $=\frac{n!}{\left(n-2\right)!}×\frac{\left(n-1\right)}{\left(n-1\right)}+\frac{2n!}{\left(n-1\right)!}$ ✓

 $=\frac{n!×\left(n-1\right)+2n!}{\left(n-1\right)!}=\frac{n!\left(n-1+2\right)}{\left(n-1\right)!}$ ✓

 $=\frac{n! ×\left(n+1\right) }{\left(n+1-2\right)! }=\frac{\left(n+1\right)!}{\left(n+1-2\right)}==$ RHS ✓ [14]

12. (a) $w=\left(\begin{matrix}-20\cos(10°)\\-20\sin(10°)\end{matrix}\right)$

 Hovering speed $=-w=\left(\begin{matrix}20\cos(10°)\\20\sin(10°)\end{matrix}\right)$ ✓✓

 (b) (i)



✓ wind vector with correct x-axis

 angle

✓ drone vector as a side of

 parallelogram and angle θ

✓ resultant vector pointing towards O,

 and diagonal of parallelogram

 (ii) $w=\left(\begin{matrix}-20\cos(10°)\\-20\sin(10°)\end{matrix}\right) d=\left(\begin{matrix}-25\cos(θ)\\25\sin(θ)\end{matrix}\right) r=\left(\begin{matrix}-r\cos(24.23°)\\r\sin(24.23)\end{matrix}\right)$ ✓✓✓

 (iii) $\begin{matrix}-r\cos(24.23°)&= -20\cos(10°)&-25\cos(θ)\\r\sin(24.23°)&= -20\sin(10°)&25\sin(θ)\end{matrix}$

 $∴ \begin{matrix}25^{2}cos^{2}θ&=&\left(r\cos(24.23°+20\cos(10°))\right)^{2}\\25^{2}sin^{2}θ&=&\left(r\sin(24.23°)+20\sin(10°)\right)^{2}\end{matrix}$

 $\rightarrow 25^{2}=\left(r\cos(24.23°)+20\cos(10°)\right)^{2}+\left(r\sin(24.23°)+20\sin(10°)\right)^{2}$ ✓✓

 CAS $\rightarrow r=38.8621$ m/s OR $r=-5.7897$ m/s ✓

 $\rightarrow θ=46.64°$ OR $θ=2.94°$ ✓

 $∴time=\frac{d}{v}=\frac{\sqrt{1200^{2}+540^{2}}}{38.8621}=33.86$ seconds ✓

 bearing $=270°+θ=308.86°T$ ✓ [14]

13. (a) $\sum\_{}^{}F\_{y}=300\sin(62°)+252\sin(56°)$ ✓

 $=473.8N$ ✓

 Since $473.8N<500N$ the machinery is not moving upwards ✓

 (b) No horizontal component needed ⇒ $\sum\_{}^{}F\_{x}=0$

 $∴400\cos(62°)=x\cos(56°)$ ✓

 $\rightarrow x=\frac{400\cos(62°)}{\cos(56°)}=335.82N$ ✓

 (c) $\sum\_{}^{}F\_{y}=400\sin(62°)+335.82\sin(56°)$ ✓

 $=631.59N$ ✓ [7]

14. (a) (i) Let $n\in N$ with $n=2k+1=$ odd ✓

 Then $n^{2}+1=\left(2k+1\right)^{2}+1$

 $=4k^{2}+4k+2$ ✓

 $=2\left(2k^{2}+2k+1\right)$ ✓

 Since 2 is a factor, then $n^{2}+1$ is divisible by 2, and

 hence the conjecture is true $∀ n\in N$ ✓

 (ii) Contrapositive statement:

 “if $n^{2}+1$ is odd, then $n$ is even.” ✓

 Let $n^{2}+1=odd=2k+1$

 $∴n^{2}=2k$ ✓

 $⇒n^{2}=even ⇒ n=even$ ✓

 Since the contrapositive statement is true $∀ n\in N$,

 then the original conjecture is true $∀ n\in N$ ✓

 (b) A ⇒ B:

 If the quadrilateral has two diagonals that intersect at right angles,

 then the quadrilateral is a rhombus, which implies it does have

 two pairs of parallel sides.

 $∴$ A ⇒ B is true ✓

 B ⇒ A:

 If the quadrilateral has two pairs of parallel sides then it is a parallelogram,

which does not necessarily imply it is a rhombus, and therefore it does not necessarily have diagonals that intersect at right angles.

 $∴$ B ⇒ A is false ✓

 Therefore, A ⇔ B is a false statement. ✓

(c) Assume that $n$ is odd and $n^{2}$ is even. ✓

 Then $∃ k\in N: n=2k+1$

 $\rightarrow n^{2}=\left(2k+1\right)^{2}=4k^{2}+4k+1 $ ✓

 $=2\left(2k^{2}+2k\right)+1=2m+1=odd$ ✓

 Since $n^{2}$ is both even and odd simultaneously, this is a contradiction ✓

 and therefore the original conjecture must be true $∀ n\in N, n$ even. [15]

15. (a) (i) $\left|AB\right|^{2}=\left|OA\right|^{2}+\left|OB\right|^{2}-2\left|OA\right|\left|OB\right|\cos(θ)$ ✓

 (ii) $\vec{AB}∙\vec{AB}=\left|OA\right|^{2}+\left|OB\right|^{2}-2\left|OA\right|\left|OB\right|\cos(θ)$

 LHS $=\left(\vec{OB}-\vec{OA}\right)∙\left(\vec{OB}-\vec{OA}\right)$ ✓

 $=\vec{OB}∙\vec{OB}-\vec{OB}∙\vec{OA}-\vec{OA}∙\vec{OB}+\vec{OA}∙\vec{OA}$ ✓

 $=\left|OB\right|^{2}+\left|OA\right|^{2}-2 \vec{OA}∙\vec{OB}$ ✓

 $∴\left|OB\right|^{2}+\left|OA\right|^{2}-2 \vec{OA}∙\vec{OB}=\left|OA\right|^{2}+\left|OB\right|^{2}-2\left|OA\right|\left|OB\right|\cos(θ)$

 $\rightarrow -2 \vec{OA}∙\vec{OB}=-2\left|OA\right|\left|OB\right|\cos(θ)$ ✓

 $\rightarrow \vec{OA}∙\vec{OB}=\left|OA\right|\left|OB\right|\cos(θ)$ as required

 (b) (i) $\left(\begin{matrix}4\\5\end{matrix}\right)∙\left(\begin{matrix}2\\-3\end{matrix}\right)=\left|\begin{matrix}4\\5\end{matrix}\right|×\left|\begin{matrix}2\\-3\end{matrix}\right|\cos(θ)$

 $8-15=\sqrt{41}×\sqrt{13}\cos(θ)$ ✓

 $∴\cos(θ)=-\frac{7}{\sqrt{533}}$ ✓

 Since $\cos(θ)<0$ ⇒ $θ$ is obtuse ✓



 (ii) ⇒ $\sin(θ)=\frac{22}{\sqrt{533}}$ ✓

 $∴$ area ΔOAB $=\frac{1}{2}\left|OA\right|\left|OB\right|\sin(θ)$

 $=\frac{1}{2} \sqrt{41}×\sqrt{13}×\frac{22}{\sqrt{533}}=11$ units2 ✓ [10]

16. P, Q, R and S are the midpoints of their respective sides:

 $\vec{OP}=\left(\begin{array}{c}-0.5\\4\end{array}\right) \vec{OQ}=\left(\begin{array}{c}6\\1.5\end{array}\right) \vec{OR}=\left(\begin{array}{c}1.5\\-4\end{array}\right) \vec{OS}=\left(\begin{array}{c}-5\\-1.5\end{array}\right)$ ✓

 Therefore:

 $\vec{PQ}=\left(\begin{array}{c}6\\1.5\end{array}\right)-\left(\begin{array}{c}-0.5\\4\end{array}\right)=\left(\begin{array}{c}6.5\\-2.5\end{array}\right)$

 $\vec{SR}=\left(\begin{array}{c}1.5\\-4\end{array}\right)-\left(\begin{array}{c}-5\\-1.5\end{array}\right)=\left(\begin{array}{c}6.5\\-2.5\end{array}\right)$ ✓

 $\vec{PQ}=\vec{SR} ⇒ ∴ \vec{PQ}$ // $\vec{SR}$ ✓

 $\vec{PS}=\left(\begin{array}{c}-5\\-1.5\end{array}\right)-\left(\begin{array}{c}-0.5\\4\end{array}\right)=\left(\begin{array}{c}-4.5\\-5.5\end{array}\right)$

 $\vec{QR}=\left(\begin{array}{c}1.5\\-4\end{array}\right)-\left(\begin{array}{c}6\\1.5\end{array}\right)=\left(\begin{array}{c}-4.5\\-5.5\end{array}\right)$ ✓

 $\vec{PS}=\vec{QR} ⇒ ∴ \vec{PS}$ // $\vec{QR}$ ✓

 Since $\vec{PQ}$ // $\vec{SR}$ and $\vec{PS}$ // $\vec{QR}$ ⇒ $PQRS$ is a parallelogram [5]